Topological interactions of Higgs bosons

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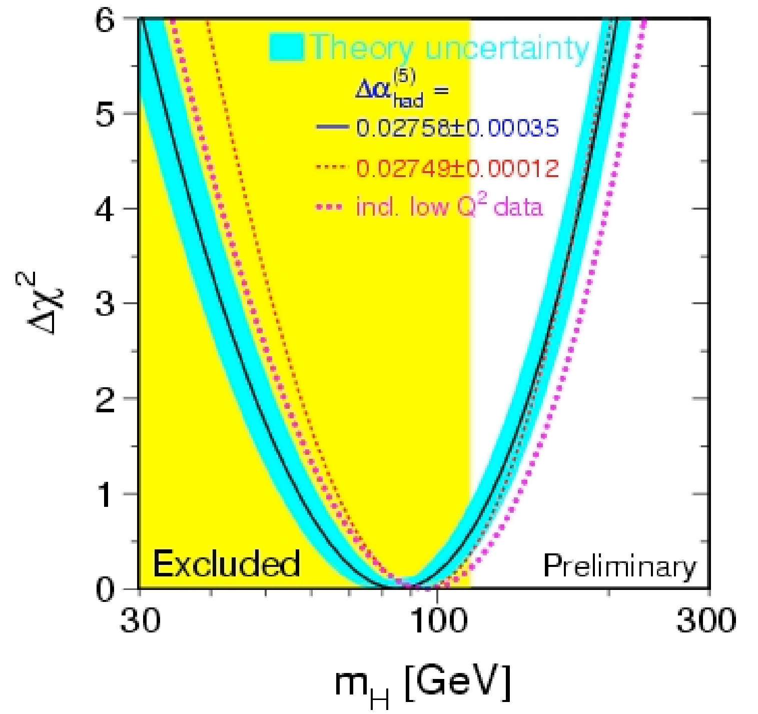
Reference:

C.T. Hill and R.J. Hill, hep-ph/0701044
(to appear in PRD)

Questions of Phenomenology

Question 1: Does a Higgs boson exist ?

- So far, precision measurements are consistent with a standard model electroweak sector described, at least effectively, by a scalar Higgs field < 200 GeV



[LEP electroweak WG]

- Higgs field gives an economical, if not particularly insightful, description of fermion masses

Question 2: Is electroweak symmetry broken by fermion condensation ?

- So far (neglecting gravity) all observed particles follow the pattern of spin-1/2 fermions interacting by exchange of spin-1 gauge fields

- SU(2)xU(1) *is* broken in the standard model without a Higgs field, by the strong interaction
(But m_W is O(1 GeV), not O(100 GeV))

Question 1+2: Is there a composite Higgs boson ?

the above motivations, and

- Composite particles, like pions, kaons, eta, can be naturally light compared to the scale of other new physics

- If not the most "exciting" scenario, at least not preposterous. Not ruled out, and an obvious benchmark at the LHC (versus SUSY, extra dimensions, ...)

Can start at the top, and work down:

>100 TeV ? “extended supercolor”

10 TeV ? “supercolor” scale

How can a large separation be explained ?

0.1 TeV weak scale

Or start at the bottom and work up:
simply ask: “suppose we see some NGB’s - then what?”

Questions of Theory

- *Question 1*: What is the most general four-dimensional effective Lagrangian describing Nambu-Goldstone bosons (NGBs) of a broken symmetry ?

Logically distinct from, but (deeply) related to:

- *Question 2*: What is the low energy description of a theory of strongly coupled fermions ?

Outline

- A simple WZ term for a simple little Higgs model
- Anomalies and Wess-Zumino-Witten terms
- Some implications for Little Higgs theories

A simple model

Symmetry breaking G → H

G=SU(3), H=SU(2)

Why?

- Fermion theories have SU(n_f) symmetry (although we don't have to talk about fermions)
- Need at least SU(2) electroweak unbroken

⇒ Consider SU(3)→SU(2)

Which SU(2) is unbroken?

- $\lambda^2\ \lambda^5\ \lambda^7$:

$$\begin{pmatrix} \cdot & -i & \cdot \\ i & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \quad \begin{pmatrix} \cdot & \cdot & -i \\ \cdot & \cdot & \cdot \\ i & \cdot & \cdot \end{pmatrix}, \quad \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & -i \\ \cdot & i & \cdot \end{pmatrix}$$

Five broken generators don't arrange into doublet + singlet

- $\lambda^1\ \lambda^2\ \lambda^3$:

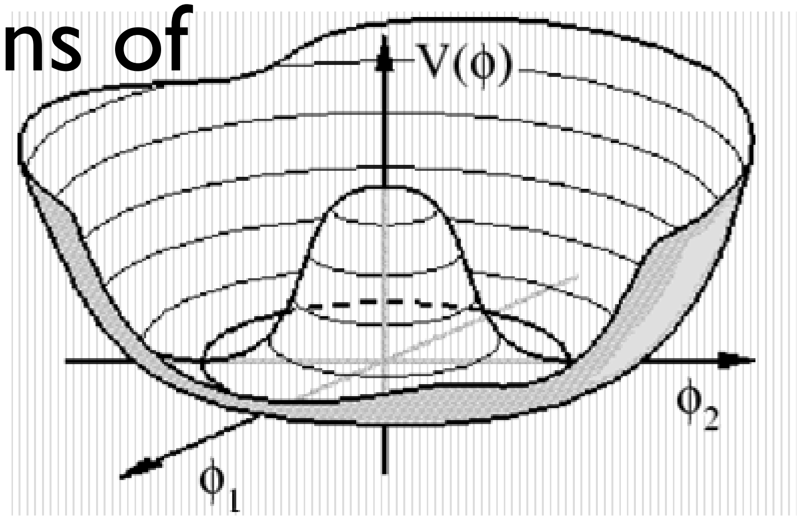
$$\begin{pmatrix} \cdot & 1 & \cdot \\ 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \quad \begin{pmatrix} \cdot & -i & \cdot \\ i & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \quad \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

NGB's: a doublet and a singlet

Effective actions for NGBs

(≈ chiral Lagrangians, but haven't said anything about fermions)

Field space M = modes along flat directions of degenerate vacua



Start with a given vacuum state: $|0\rangle$

Act on vacuum with general G group element: $|0'\rangle = g|0\rangle$

If all generators are broken: $M = \{g|0\rangle, g \in G\} \leftrightarrow G$

If generators of a subgroup H not broken: $gh|0\rangle = g|0\rangle$

$$M \leftrightarrow G/H = \{[g], g \in G\}\,, \quad [g] = \{gh, h \in H\}$$

Examples: $SU(3) \times SU(3) \times U(1)/SU(3) \times U(1) \leftrightarrow SU(3)$

$$SU(3)/SU(2) \leftrightarrow S^5$$

Our field space for SU(3)/SU(2) is the five-sphere

$$\Phi = \begin{pmatrix} \phi^1 + i\phi^2 \\ \phi^3 + i\phi^4 \\ \phi^5 + i\phi^6 \end{pmatrix} \qquad \Phi^\dagger \Phi = \sum_{i=1}^{6} (\phi^i)^2 = 1$$

Local choice of coordinates:

$$\Phi = \exp\left[i \begin{pmatrix} \eta & \cdot & \\ \cdot & \eta & H \\ & H^\dagger & -2\eta \end{pmatrix} \right] \begin{pmatrix} \cdot \\ \cdot \\ 1 \end{pmatrix}$$

Identify unbroken group with electroweak

$$A = \begin{pmatrix} W & \cdot \\ \cdot & \cdot \end{pmatrix} \qquad H \to e^{i\epsilon_W} H\,, \quad \eta \to \eta$$

What is the most general action for the dynamics of W, H, η ?

Must be electroweak gauge-invariant:

$$(F_W^{\mu\nu})^2\,,|D_\mu H|^2\,,(\partial_\mu\eta)^2\,,\partial^\mu\eta H^\dagger D_\mu H\,,\epsilon^{\mu\nu\rho\sigma}\eta F_{\mu\nu}F_{\rho\sigma}\,,\ldots$$

But further constraints from symmetry - everything must be written in terms of Φ

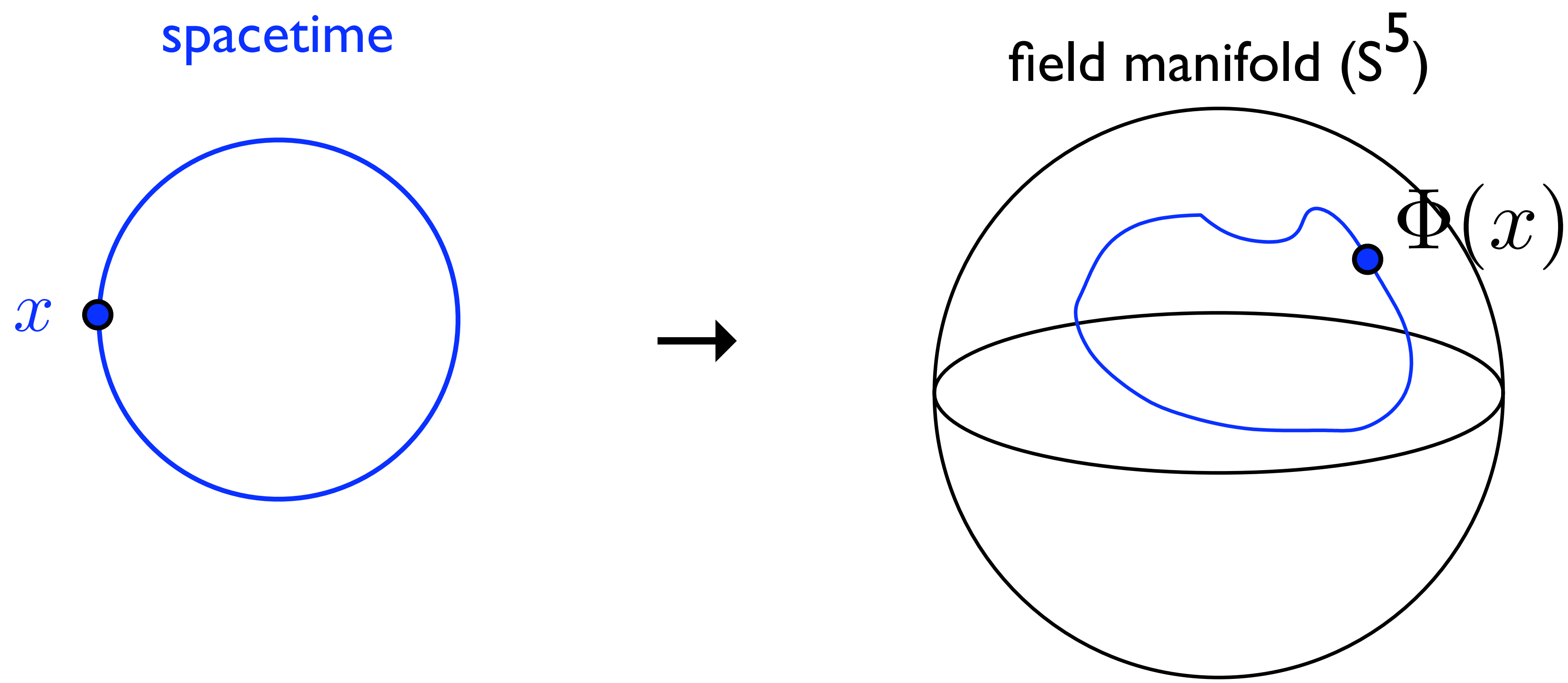
$$\Gamma(\Phi) = \int d^4x\,|\partial_\mu\Phi|^2 + c_1|\partial_\mu\Phi|^4 + c_2\Phi^\dagger\partial^4\Phi + \ldots$$

Done ? No..

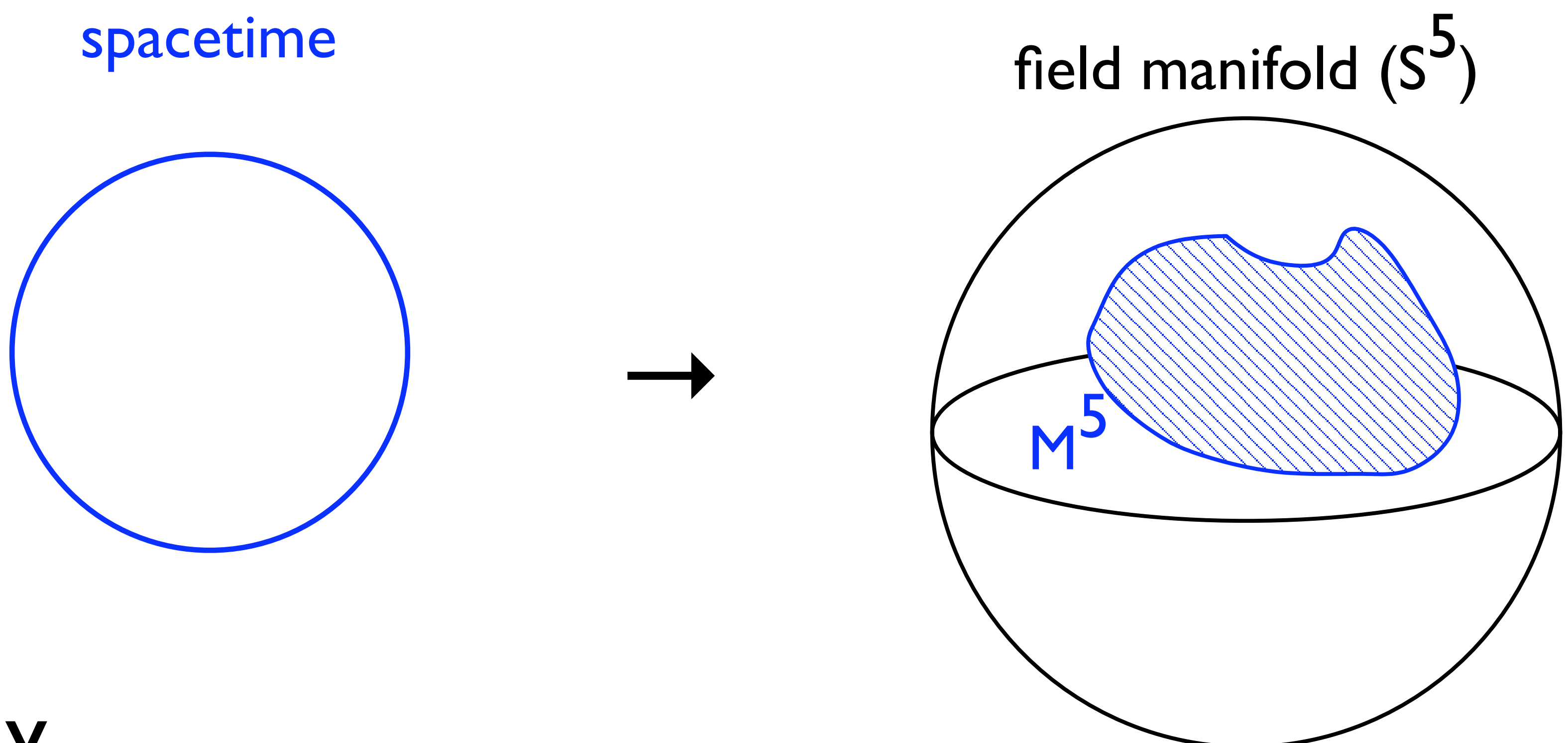
$$\Gamma'(\Phi) = \text{number} \times \text{``area bounded by the image of spacetime on } S^5\text{''}$$

Together, Γ and Γ' give the general effective action for Φ

Nothing subtle, just another way of building a local, four-dimensional, SU(3)-invariant action.



Nothing subtle, just another way of building a local, four-dimensional, SU(3)-invariant action.

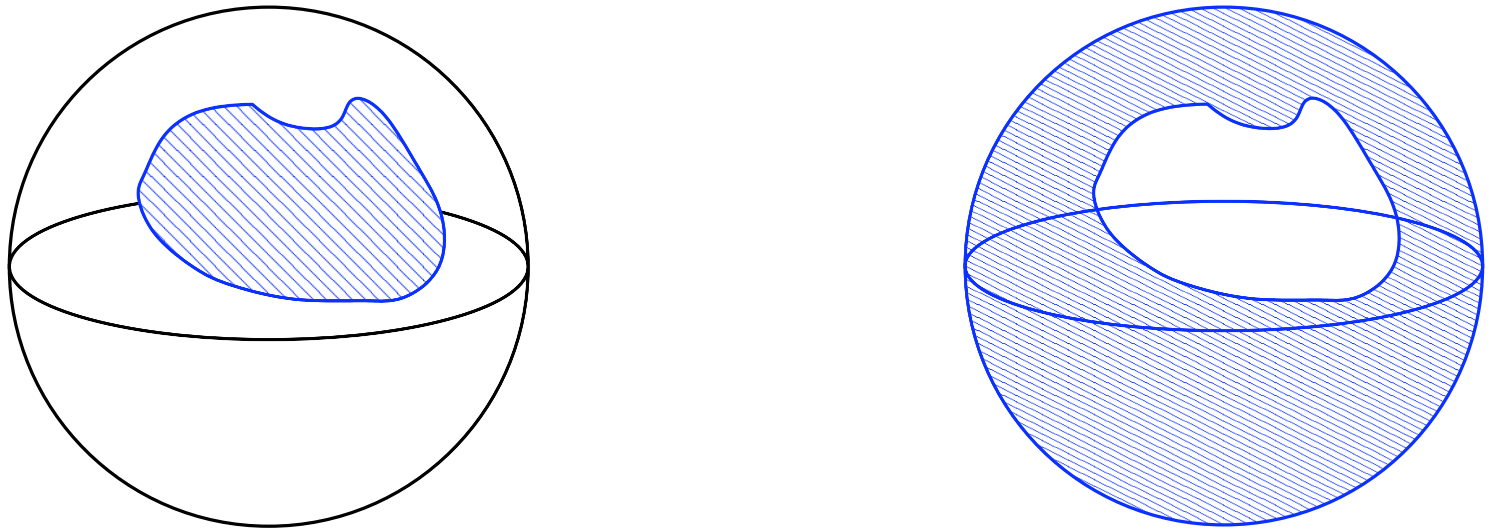


Explicitly,

$$\Gamma'(\Phi) \propto \int_{M^5} -\frac{i}{8}\Phi^\dagger d\Phi d\Phi^\dagger d\Phi d\Phi^\dagger d\Phi$$

$$\frac{1}{\sqrt{1-(\phi^1)^2-\cdots-(\phi^5)^2}}\epsilon^{ABCDE}\frac{\partial\phi^1}{\partial y^A}\frac{\partial\phi^2}{\partial y^B}\frac{\partial\phi^3}{\partial y^C}\frac{\partial\phi^4}{\partial y^D}\frac{\partial\phi^5}{\partial y^E}d^5y$$

Quantization:



Can only be consistent if difference between choices of bounding surface is 2π x integer

[Witten 1982]

[Volume of S^5] = π^3 ⇒

$$\Gamma'(\Phi) = \text{integer} \times 2\pi \times \frac{1}{\pi^3} \int_{M^5} -\frac{i}{8} \Phi^\dagger d\Phi d\Phi^\dagger d\Phi d\Phi^\dagger d\Phi$$

$$\boxed{\text{(quantized) odd parity interactions of NGBs:} \quad \Gamma'(\Phi) \propto \int d^4x \, \epsilon^{\mu\nu\rho\sigma} \eta \partial_\mu H^\dagger \partial_\nu H \partial_\rho H^\dagger \partial_\sigma H + \ldots}$$

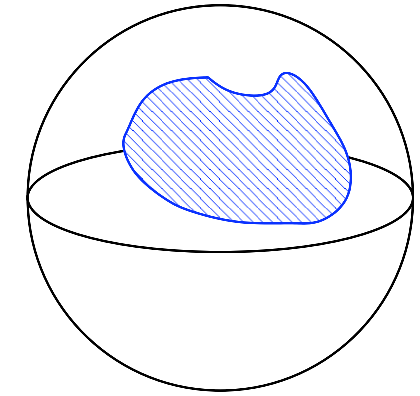
Γ' is odd under:

$$\begin{array}{ccc} \vec{x} & \rightarrow & -\vec{x} \\ \eta & \rightarrow & +\eta \\ H & \rightarrow & +H \end{array}$$

(Γ is even)

Such five-boson interactions difficult to observe directly (even in QCD). Easier to see this physics when gauge fields are coupled to the system.

Mathematical jargon



For a given smooth $\Phi(x)$, there exists a bounding surface: $\pi_4(S^5)=0$

Most perverse we can be about choosing different bounding surfaces is to wrap around the sphere: $\pi_5(S^5)=Z$

These are obvious for the present case. For $SU(n)xSU(n)\to SU(n)$, also $\pi_4(SU(n))=0$, $\pi_5(SU(n))=Z$.

$SU(n)xSU(n)/SU(n)=SU(n)$	$SU(3)/SU(2)= S^5$
algebra trivial, topology complicated	topology trivial, algebra complicated

Gauging

Recall gauge invariance of kinetic term

$$\mathcal{L}_0 = \partial_\mu \phi^\dagger \partial_\mu \phi$$

$$\delta \mathcal{L}_0 = -i\phi^\dagger \partial_\mu \epsilon \partial_\mu \phi + i\partial_\mu \phi^\dagger \partial_\mu \epsilon \phi$$

$$\mathcal{L}_1 = i\phi^\dagger A_\mu \partial_\mu \phi - i\partial_\mu \phi^\dagger A_\mu \phi$$

$$\delta(\mathcal{L}_0 + \mathcal{L}_1) = -\phi^\dagger A_\mu \partial_\mu \epsilon \phi - \phi^\dagger \partial_\mu \epsilon A_\mu \phi$$

$$\mathcal{L}_2 = \phi^\dagger A_\mu A_\mu \phi$$

$$\delta(\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2) = 0$$

$$\mathcal{L} = (\partial_\mu \phi^\dagger + i\phi^\dagger A_\mu)(\partial_\mu \phi - iA_\mu \phi)$$

Play the same game with the topological term

$$\Gamma_0 = \frac{-ip}{4\pi^2}\int_{M^5} \Phi^\dagger d\Phi (d\Phi^\dagger d\Phi)^2$$

$$\delta\Gamma_0 = \frac{p}{4\pi^2}\int_{M^5} d\left[\Phi^\dagger\epsilon\Phi(d\Phi^\dagger d\Phi)^2 + 2\epsilon^A d(\Phi^\dagger\lambda^A\Phi)\Phi^\dagger d\Phi d\Phi^\dagger d\Phi\right] - 3\epsilon^A d(\Phi^\dagger\lambda^A\Phi)(d\Phi^\dagger d\Phi)^2$$

Doesn't look four-dimensional !

$$d(\Phi^\dagger\lambda^A\Phi)(d\Phi^\dagger d\Phi)^2 = 0$$

$$\delta\Gamma_0 = \frac{p}{4\pi^2}\int_{M^4} \left[\Phi^\dagger\epsilon\Phi(d\Phi^\dagger d\Phi)^2 + 2\epsilon^A d(\Phi^\dagger\lambda^A\Phi)\Phi^\dagger d\Phi d\Phi^\dagger d\Phi\right]$$

Doesn't look globally invariant !

$$\Phi^\dagger\lambda^A\Phi(d\Phi^\dagger d\Phi)^2 + 2d(\Phi^\dagger\lambda^A\Phi)\Phi^\dagger d\Phi d\Phi^\dagger d\Phi = -2d\Phi^\dagger\lambda^A d\Phi d\Phi^\dagger d\Phi$$

$$\delta\Gamma_0 = \frac{p}{4\pi^2}\int_{M^4} \left(\Phi^\dagger d\epsilon d\Phi + d\Phi^\dagger d\epsilon\Phi\right) d\Phi^\dagger d\Phi$$

$$\begin{aligned}\Gamma_1 \quad &= \quad \frac{p}{4\pi^2}\int_{M^4} \left(-\Phi^\dagger A d\Phi - d\Phi^\dagger A\Phi\right) d\Phi^\dagger d\Phi \\ &\quad + c_1\Phi^\dagger d\Phi(\Phi^\dagger dA d\Phi - d\Phi^\dagger dA\Phi) + c_2 d\Phi^\dagger d\Phi\Phi^\dagger dA\Phi\end{aligned}$$

Who ordered the fermions ?

After adding 1,2,3,4 gauge fields, the action arranges into a set of (unquantized) gauge invariant operators, plus a (quantized) action with anomalous gauge variation:

$$\delta\Gamma_{WZW} = -\frac{2p}{24\pi^2}\int_{M^4} \mathrm{Tr}\left\{\left(\epsilon - \frac{\epsilon_0}{2}\right)\left[\left(dA - \frac{1}{2}dA_0\right)^2 - \frac{i}{2}d\left(A - \frac{1}{2}A_0\right)^3\right]\right\} + \frac{27}{8}\epsilon_0(dA_0)^2$$

Recall the fermion anomaly:

$$\delta\Gamma = -\frac{N_c}{24\pi^2}\int_{M^4} \mathrm{Tr}\left\{\epsilon_L\left[(dA_L)^2 - \frac{i}{2}d(A_L^3)\right]\right\} - (L \leftrightarrow R)$$

Constraints on the UV completion:

- anomaly for triplet in N, singlet in N-bar of SU(N)

$$\Phi \sim \Psi_L \bar{q}_R = \begin{pmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix} \bar{q}$$

- Nc=2p is even

Anomaly interactions:

$$\frac{-N}{8\pi^2\sqrt{3}} \int d^4x \, \epsilon^{\mu\nu\rho\sigma} \eta \mathrm{Tr}(F_W^{\mu\nu} F_W^{\rho\sigma}) + \ldots$$

$$d(\Phi^\dagger\lambda^A\Phi)(d\Phi^\dagger d\Phi)^2 = 0$$

Some heavy machinery for these identities:

$$\mathcal{L}_a = i_a d + d i_a$$

inner derivative

Lie derivative   exterior derivative

$$\delta\Gamma = \int \delta\omega = i\int d\epsilon_a\, i_a\omega = i\int d(\epsilon_a\, i_a\omega) - \epsilon_a \underbrace{d i_a \omega}_{\underbrace{\mathcal{L}_a\omega}_{0} - i_a \underbrace{d\omega}_{0}}$$

$$\delta\Phi = i\epsilon\Phi$$

ω globally invariant   ω closed

$$\delta\Gamma_0 = \frac{p}{4\pi^2}\int_{M^5} d\left[\Phi^\dagger\epsilon\Phi(d\Phi^\dagger d\Phi)^2 + 2d(\Phi^\dagger\epsilon\Phi)\Phi^\dagger d\Phi d\Phi^\dagger d\Phi\right] - 3\epsilon^A d(\Phi^\dagger\lambda^A\Phi)(d\Phi^\dagger d\Phi)^2$$

⇒Makes action manifestly four-dimensional (as it has to be)

$$\delta\Gamma = i\int_{M^4}\epsilon_a \underbrace{i_a\omega}_{d\theta_a} = -i\int_{M^4} d\epsilon_a\theta_a$$

$$di_a\omega = \mathcal{L}_a\omega - i_a d\omega = 0$$

$\Rightarrow$Makes variation manifestly local (as it has to be)

- Using these methods, can show that it is always possible to keep adding gauge fields until the total gauge variation of the action is independent of meson fields

[e.g. Hull and Spence 1991]

$$\delta_\epsilon\Gamma(\pi, A) \sim \int f(\epsilon, A)$$

- By definition, this is a “consistent” anomaly

An easy but important theorem:

(fermions without fermions)

Consider the NGBs from a spontaneously broken continuous symmetry.
If it is possible to write a topological action, then the symmetry breaking has a chiral fermion interpretation.

- in mathematical terms: need a closed, globally-invariant five-form
- example: SU(3)→SU(2) (a five-form on a five manifold is closed)
- example: any theory with a parity defined on the symmetry generators

Parity

Many patterns of symmetry breaking define a parity:

[V,V]~V, [V,A]~A, [A,A]~V

V→+V

A→-A

unbroken "vector" generators

broken "axial" generators

Examples:

- SU(n)xSU(n)/SU(n)

$$V^a \sim t^a\,, \quad A^a \sim t^a\gamma_5$$

- SU(n)/SO(n)

$$SU(3)/SO(3): \quad V \sim \lambda^{2,5,7}\,, \quad A \sim \lambda^{1,3,4,6,8}$$

- SU(2n)/Sp(2n)

Non-example:

- SU(3)/SU(2) $\quad V \sim \lambda^{1,2,3}\,, \quad A \sim \lambda^{4,5,6,7,8}$

At first sight, it appears that the effective action for NGB's conserves the internal parity

$$\Gamma \sim \int d^4x\, |D_\mu U|^2 + c_1 |D_\mu U|^4 + c_2 D_\mu U D_\nu U^\dagger D_\mu U D_\nu U^\dagger + \dots$$

This would forbid interactions involving odd numbers of NGB's, e.g. $\pi_0 \to \gamma\gamma$

This action also has no anomalous gauge variation, so can't be a faithful description of low-energy QCD - *too much symmetry, not enough anomaly*

In fact, whenever a parity can be defined, can construct a term in the action that breaks it..

Universal form of WZW

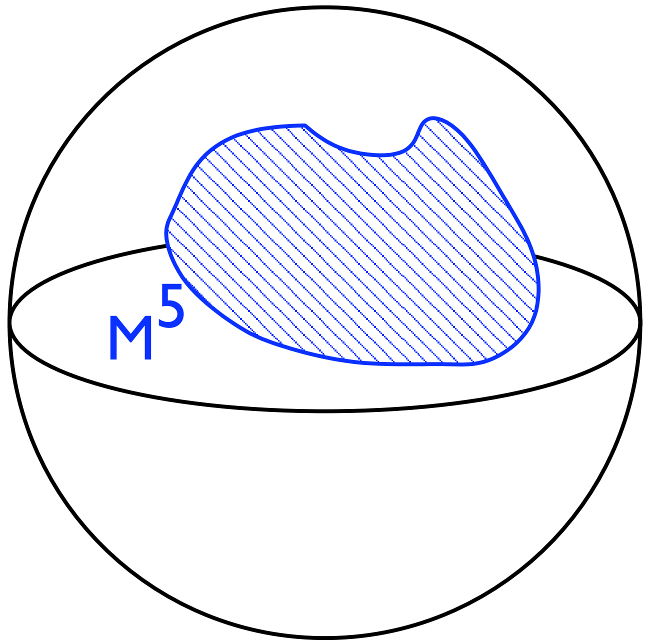
First, recall SU(n)xSU(n)/SU(n):
Just like before, but harder to visualize

$$\Gamma'(U) = \text{number} \times \text{“area bounded by the image of spacetime on } SU(N)\text{”}$$

$$\Gamma'(U) = -\frac{iN_c}{240\pi^2}\int_{M^5} \mathrm{Tr}(\alpha^5)$$

$$\alpha = (dU)\,U^\dagger$$

field manifold (SU(n))



- construction allowed by $\pi_4(SU(n))=0$, $\pi_5(SU(n))=Z$
- quantization condition necessary for consistency

Suppose the NGB's can be collected into a unitary matrix such that:

$$U \rightarrow e^{i\epsilon_L} U e^{-i\epsilon_R}$$

(e.g., for SU(n)xSU(n)/SU(n) = SU(n))

It follows automatically that we can write a closed, globally invariant, five-form:

$$\mathrm{Tr}(\alpha^5)\,, \quad \alpha = (dU)\,U^\dagger$$

This was the mathematical ingredient in the "fermions without fermions" theorem

For a general symmetry breaking pattern, the full symmetry group can be defined to act on NGB's as:

$$e^{i\pi} \longrightarrow e^{i\epsilon} e^{i\pi} e^{-i\epsilon'(\epsilon,\pi)}$$

general group element

element of unbroken group

[Coleman et.al. 1969]

Let R denote the parity: π is odd, ϵ' is even

$$e^{2i\pi'} = e^{i\pi'} R(e^{-i\pi'}) = e^{i\epsilon} e^{2i\pi} e^{-iR(\epsilon)}$$

Thus U=exp(2iπ) has the correct properties to build a topological action

Another way to proceed

Work directly from the nonlinear realization:

$$e^{i\pi} \longrightarrow e^{i\epsilon} e^{i\pi} e^{-i\epsilon'(\epsilon,\pi)}$$

View this as a restriction of the more general case with a full SU(n) multiplet of NGBs in π, and unconstrained variations ϵ_L and ϵ_R:

$$e^{i\pi} \longrightarrow e^{i\epsilon_L} e^{i\pi} e^{-i\epsilon_R}$$

This has too many NGBs, and too many symmetry generators

Eat and Decouple:

eat the extra pions with strongly coupled gauge field for ϵ'

$$\Gamma \sim \int d^4x \, \{-(F_L^{\mu\nu})^2 - (F_R^{\mu\nu})^2 + |D_\mu U|^2\}$$

$$D^\mu U = \partial^\mu U - i g_L A_L^\mu U + i U g_R A_R^\mu$$

Take $g_R \to \infty$, yielding e.o.m. for A_R as a function of A_L, U

To see how this works in a specific example, return to SU(3)/SU(2):

$$A_R^\mu = \left\langle U^\dagger (A_L^\mu + i\partial^\mu) U \right\rangle_{SU(2)}$$

- What do we get when we substitute this solution for A_R into the SU(3)xSU(3)/SU(3) WZW term ?
- Is it the same as the direct construction from SU(3)/SU(2)?

First, they have the same anomaly:

$$\delta\Gamma = -\frac{N}{24\pi^2}\int_{M^4} \mathrm{Tr}\left\{\epsilon_L\left[(dA_L)^2 - \frac{i}{2}d(A_L^3)\right]\right\} - \underbrace{\mathrm{Tr}\left\{\epsilon_R\left[(dA_R)^2 - \frac{i}{2}d(A_R^3)\right]\right\}}_{0}$$

note: appears that (even or odd) integer is possible - odd ruled out by global SU(2) anomaly

Can compare order by order in pions: the same

note: more gauge invariant operators appear in the direct construction (Φ) than from the nonlinear realization (U) - from a U can get a Φ, but not vice-versa

Equivalent spinor field theories

To establish "equivalence" between different fermion field theories, a necessary condition is that, if chiral symmetry breaking happens, the resulting chiral Lagrangians agree

- If two chiral Lagrangians generate the same anomaly, are they the same (up to gauge-invariant operators) ?

- Or, given a (consistent) anomaly, can we recreate the action (up to gauge invariant operators) ?

Integrating a consistent anomaly

Simpler case: consider a U(1) field φ=exp(iπ), coupled to a photon. Evaluate the action at φ=1:

$$\Gamma = \int d^4x\,(\partial\phi^* + iA\phi^*)(\partial\phi - iA\phi) \to \int d^4x\, A^2 = \Gamma\big|_{\pi=0}$$

Now take a local variation of the new action:

$$\delta\Gamma' = \int d^4x 2\partial\epsilon\, A = \int d^4x\,\epsilon\,(-2\partial A) \equiv \int d^4x\epsilon\,\mathcal{A}(A)$$

$$\delta A = \partial\epsilon$$

"consistent" anomaly

Given this "consistent" anomaly, can we build a boson theory that generates it?

“integrate” the anomaly:

$$\Gamma' = \int_0^1 dt \int d^4x\, \pi(x)\, \underbrace{\mathcal{A}(\underbrace{e^{-it\pi}(A+i\partial)e^{it\pi}}_{A-t\partial\pi})}_{-2\partial(A-t\partial\pi)}$$

$$= \int d^4x \; -2\pi\partial A + \pi\partial^2\pi$$

Add the “boundary” condition:

$$\Gamma\big|_{\pi=0} = \int d^4x A^2$$

Recover the original action:

$$\Gamma = \Gamma\big|_{\pi=0} + \Gamma' = |(\partial - iA)e^{i\pi}|^2$$

Apply to chiral Lagrangians

In general, if we have a consistent anomaly:

$$\delta\Gamma = \int d^4x\, \epsilon^a(x)\mathcal{A}^a(A)$$

which vanishes for "*a*" in a subgroup H, then we can integrate to obtain an action for SU(n)/H:

$$\Gamma = \int_0^1 dt \int d^4x\, \pi^a(x)\mathcal{A}^a\left(e^{-it\pi}(A+i\partial)e^{it\pi}\right)$$

[e.g. Zumino et.al. 1996]

- If two chiral Lagrangians generate the same anomaly, then they are the same (up to gauge-invariant operators)

- Interesting equivalences between WZ actions obtained from different underlying fermion theories

Ingredients of a little higgs model

(1) Find a mechanism for a light "Higgs" field to leak down below a "supercolor" scale (~10 TeV)

(2) Generate a Higgs potential, and coupling to fermions, without upsetting (1)

- concentrate on (1)
- understand (1) $\Rightarrow$ shed some light on (2)

Radiative mass corrections

consider a collection of gauged generators:

$$\Lambda = \underbrace{\Lambda_V}_{\text{unbroken}} + \underbrace{\Lambda_A}_{\text{broken}}$$

one-loop contribution to scalar masses:

$$m^2_{ab} = \underbrace{M^2}_{\text{positive}} \sum_{\Lambda} \mathrm{Tr}\left\{ \underbrace{[\Lambda_V, [\Lambda_V, t^a_A]] t^b_A}_{\substack{\text{positive and nonzero} \\ \text{if NBG charged}}} - [\Lambda_A, [\Lambda_A, t^a_A]] t^b_A \right\}$$

[e.g. Peskin 1980]

- "little higgs": to keep modes massless, gauged generators must have both unbroken and broken components
- "composite higgs": EW-symmetric vacuum can be destabilized by gauging broken generators strongly enough

Anomaly physics of little Higgs bosons

Why it's not easy to see the anomaly interactions:

Recall the case of QCD - what are the anomaly interactions involving kaons ?

- Single K interactions ruled out (isospin)
- $K^\dagger K$ interactions ruled out (parity)

Interactions must involve either other NGB's (π,η) or gauge fields for broken generators

Example: two copies of SU(3)/SU(2): Kaplan-Schmaltz

[Kaplan and Schmaltz 2003]

- Gauge both copies with the *same* SU(3) gauge field
- One copy of (H,η) eaten, one copy survives

$$\Phi = \exp\left[i \begin{pmatrix} \eta & \cdot & H \\ \cdot & \eta & \\ H^\dagger & & -2\eta \end{pmatrix} \right] \begin{pmatrix} \cdot \\ \cdot \\ 1 \end{pmatrix} \qquad A = \begin{pmatrix} W & C \\ C^\dagger & 0 \end{pmatrix}$$

Topological interactions involving H:

$$\frac{N}{16\pi^2} \int \left[(DH^\dagger)(dW - iW^2)C - C^\dagger(dW - iW^2)DH \right]$$

$$e^+e^- \to Z^* \to HC$$

Example: SU(5)/SO(5)

[Arkani-Hamed et.al. 2002]

- Collect NGBs into $\Sigma = \exp(2i\pi)$

$$\pi = \begin{pmatrix} \chi^T + \frac{1}{2}\eta & H^* & \phi^\dagger \\ H^T & -2\eta & H^\dagger \\ \phi & H & \chi + \frac{1}{2}\eta \end{pmatrix}$$

- Gauge electroweak, and eaters of χ, η

$$A = \begin{pmatrix} -W^T - \frac{1}{2}B + W'^T + \frac{1}{2}B' & \cdot & \cdot \\ \cdot & -2B' & \cdot \\ \cdot & \cdot & W + \frac{1}{2}B + W' + \frac{1}{2}B' \end{pmatrix}$$

- H, χ, η massless through one loop, φ not

Topological interactions most dramatic for anomaly-dominated decays - e.g. decay of "lightest T-odd" particle into standard model particles

Aside on parity:

Recall in QCD, there is only one parity:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + A\!\!\!/_V + A\!\!\!/_A \gamma_5)\psi$$

$$\begin{array}{ccc} \psi & \rightarrow & \gamma^0\psi \\ A_V & \rightarrow & +A_V \\ A_A & \rightarrow & -A_A \end{array} \qquad \textit{and} \qquad \vec{x} \quad \rightarrow \quad -\vec{x}$$

• leading term in chiral Lagrangian respects two parities: π→-π, x→-x

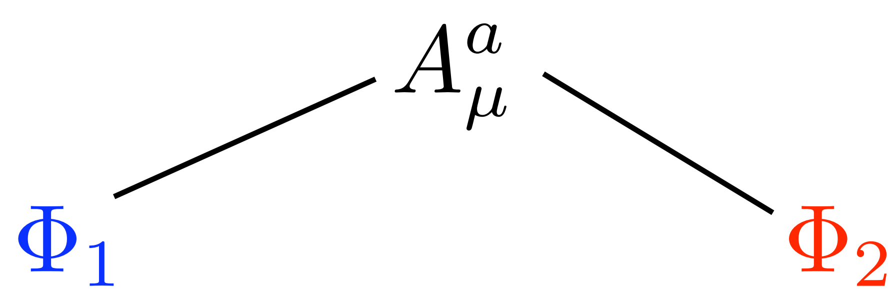
• WZW term breaks both parities, preserving only the combination

Implication for little higgs models:

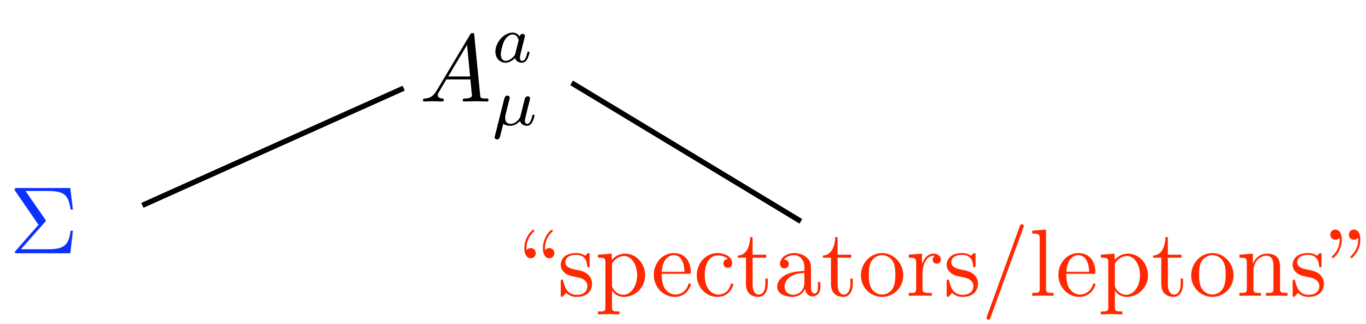
• can’t use NGB parity (“T parity”) to argue stability of lightest odd particle (unless p=0)

Recall that to implement the "little higgs" one-loop mass cancellation, need to gauge broken generators, which introduce anomalies

- cancel with decoupled sectors (e.g. Kaplan Schmaltz)



- cancel with spectators (e.g. SU(5)/SO(5))



What to do with spectators ?

An amusing QCD example

- suppose we knew the basic structure of the lepton sector, but not it's coupling to hypercharge
- suppose we knew the weak couplings of mesons, but not the number of colors

Program:

- measure N_c via anomaly ($\pi_0 \to \gamma\gamma$)
- deduce the hypercharge couplings of "spectator" leptons:

	T^3	Y
ν_L	$\frac{1}{2}$	$-\frac{1}{6}N_c$
e_L	$-\frac{1}{2}$	$-\frac{1}{6}N_c$
ν_R	0	$-\frac{1}{6}N_c + \frac{1}{2}$
e_R	0	$-\frac{1}{6}N_c - \frac{1}{2}$

Historical perspective

Technicolor/ composite higgs

- electroweak symmetry broken by fermion condensation

Extra dimension, deconstruction

- UV completion involves deconstructed extra dimension

?

Little higgs

- work directly from symmetries, leave UV completion unspecified

Even if fermions aren't mentioned by name, the anomaly physics is still present

Summary

Theory

- Simple (simplest?) WZ term: SU(3)/SU(2)

- Equivalent approaches to topological action give insight on equivalences of different fermion theories

Phenomenology

- A benchmark scenario for a nonstandard Higgs

- "Little Higgs" versus "Composite Higgs" emphasizes symmetries, but anomaly physics enters regardless and can't be neglected

- Understanding topological interactions is necessary to answer the question: " what is the higgs particle ? "